

AS Level Mathematics B (MEI)

H630/02 Pure Mathematics and Statistics

Sample Question Paper

Date – Morning/Afternoon

Time allowed: 1 hour 30 minutes

You must have:

- Printed Answer Booklet

You may use:

- a scientific or graphical calculator



INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION

- The total number of marks for this paper is **70**.
- The marks for each question are shown in brackets [].
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **8** pages.

Formulae AS level Mathematics B (MEI) (H630)

Binomial series

$$(a+b)^n = a^n + {}^n C_1 a^{n-1}b + {}^n C_2 a^{n-2}b^2 + \dots + {}^n C_r a^{n-r}b^r + \dots + b^n \quad (n \in \mathbb{N}),$$

$$\text{where } {}^n C_r = {}_n C_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots \quad (|x| < 1, n \in \mathbb{R})$$

Differentiation from first principles

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Sample variance

$$s^2 = \frac{1}{n-1} S_{xx} \text{ where } S_{xx} = \sum (x_i - \bar{x})^2 = \sum x_i^2 - \frac{(\sum x_i)^2}{n} = \sum x_i^2 - n\bar{x}^2$$

Standard deviation, $s = \sqrt{\text{variance}}$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = r) = {}^n C_r p^r q^{n-r}$ where $q = 1 - p$

Mean of X is np

Kinematics

Motion in a straight line

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u + v)t$$

$$v^2 = u^2 + 2as$$

$$s = vt - \frac{1}{2}at^2$$

Answer **all** the questions

1 Find $\int \left(x^2 + \frac{1}{x^2} \right) dx$. [3]

2 (i) Express $2\log_3 x + \log_3 a$ as a single logarithm. [1]

(ii) Given that $2\log_3 x + \log_3 a = 2$, express x in terms of a . [3]

3 Show that the area of the region bounded by the curve $y = 3x^{-\frac{3}{2}}$, the lines $x = 1$, $x = 3$ and the x -axis is $6 - 2\sqrt{3}$. [5]

4 There are four human blood groups; these are called O, A, B and AB. Each person has one of these blood groups. The table below shows the distribution of blood groups in a large country.

Blood group	Proportion of population
O	49%
A	38%
B	10%
AB	3%

Two people are selected at random from this country.

(i) Find the probability that at least one of them has blood group O. [2]

(ii) Find the probability that they have different blood groups. [3]

5 A triangular field has sides of length 100 m, 120 m and 135 m.

(i) Find the area of the field. [5]

(ii) Explain why it would not be reasonable to expect your answer in (i) to be accurate to the nearest square metre. [1]

- 6 (i) The graph of $y = 3\sin^2 \theta$ for $0^\circ \leq \theta \leq 360^\circ$ is shown in Fig. 6. On the copy of Fig. 6, sketch the graph of $y = 2\cos \theta$ for $0^\circ \leq \theta \leq 360^\circ$. [2]

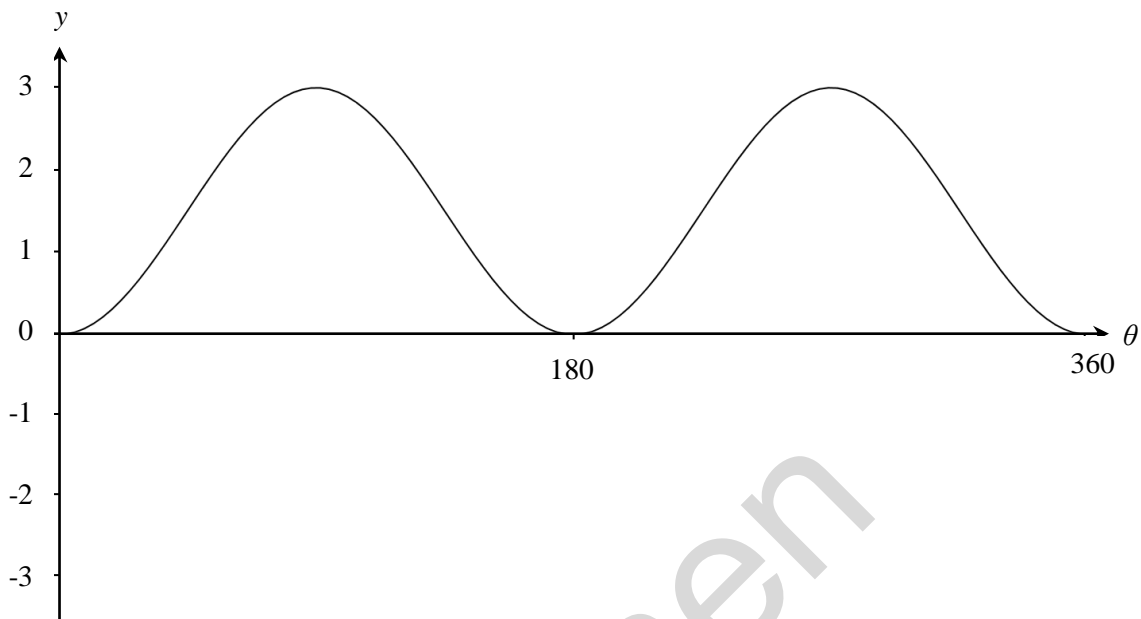


Fig. 6

- (ii) In this question you must show detailed reasoning.

Determine the values of θ , $0^\circ \leq \theta \leq 360^\circ$, for which the two graphs cross. [6]

- 7 An apple farmer has 200 apple trees. She is investigating the masses of the crops of apples from individual trees. She decides to select a sample of these trees and find the mass of the crop for each tree.

- (i) Explain how she can select a random sample of 10 different trees from the 200 trees. [2]

The masses of the crops from the 10 trees, measured in kg, are recorded as follows.

23.5 27.4 26.2 29.0 25.1 27.4 26.2 28.3 38.1 24.9

- (ii) For these data find

- the mean,
- the sample standard deviation. [2]

- (iii) Show that there is one outlier at the upper end of the data. How should the farmer decide whether to use this outlier in any further analysis of the data? [3]

- 8 In an experiment, the temperature of a hot liquid is measured every minute. The difference between the temperature of the hot liquid and room temperature is D °C at time t minutes.

Fig. 8 shows the experimental data.

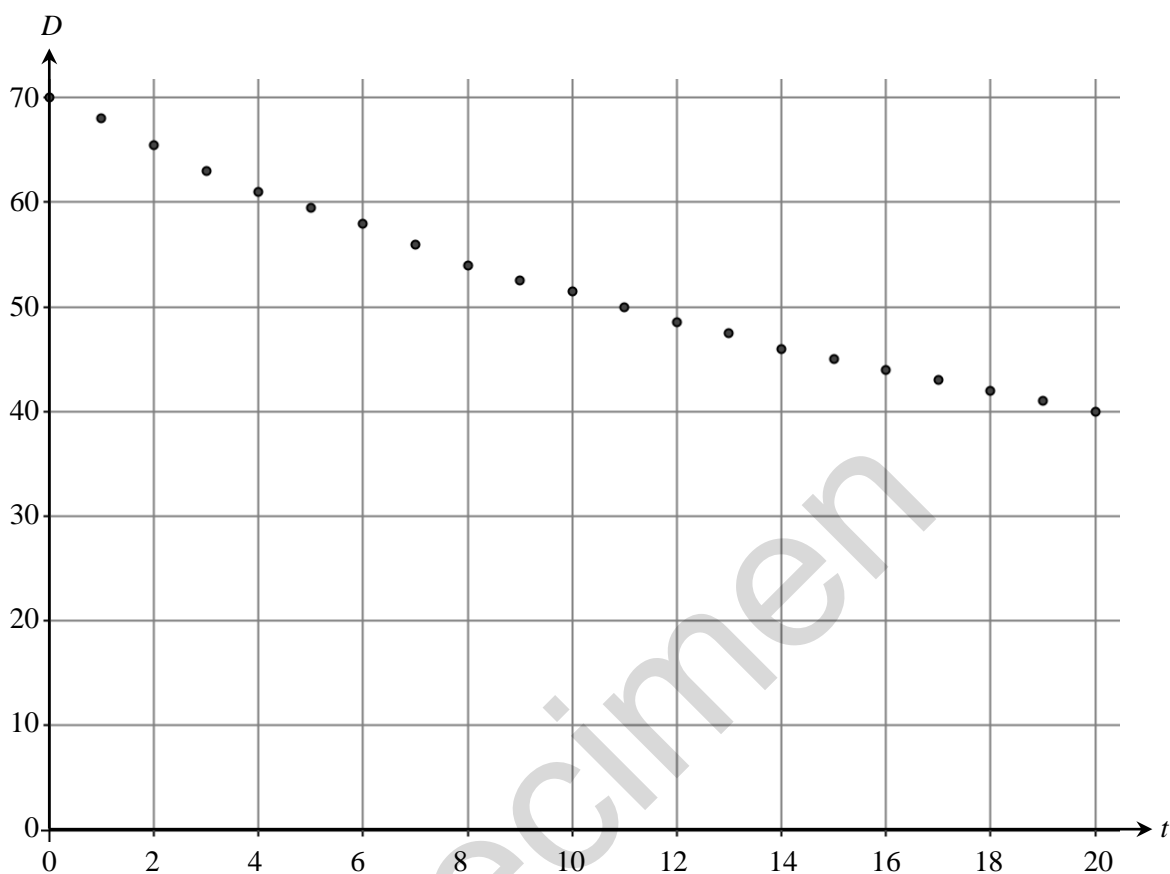


Fig. 8

It is thought that the model $D = 70e^{-0.03t}$ might fit the data.

- (i) Write down the derivative of $e^{-0.03t}$. [1]
- (ii) Explain how you know that $70e^{-0.03t}$ is a decreasing function of t . [1]
- (iii) Calculate the value of $70e^{-0.03t}$ when
- (A) $t = 0$, [1]
- (B) $t = 20$. [1]
- (iv) Using your answers to parts (ii) and (iii), discuss how well the model $D = 70e^{-0.03t}$ fits the data. [3]

- 9 The box and whisker diagrams in Fig. 9.1 summarise the birth rates per 1000 people for all the countries in three of the regions as given in the pre-release data set. They were drawn as part of an investigation comparing birth rates in different regions of the world.

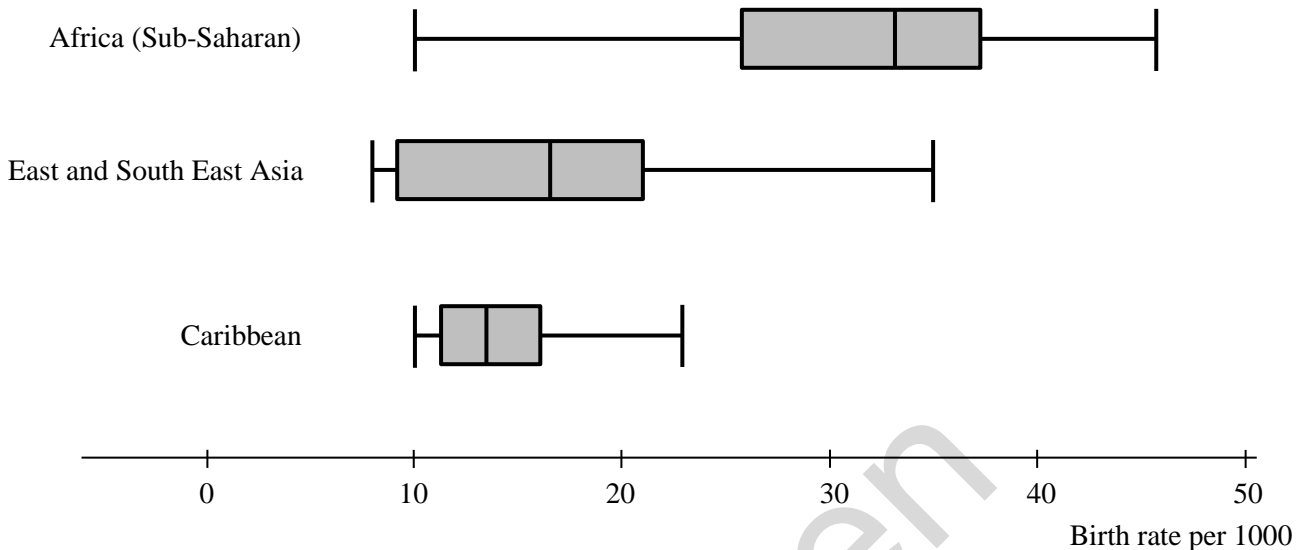


Fig. 9.1

- (i) Discuss the distributions of birth rates in these regions of the world. Make three different statements. You should refer to **both** information from the box and whisker diagrams **and** your knowledge of the large data set. [3]
- (ii) The birth rates for all the countries in Australasia are shown below.

Country	Birth rate per 1000
Australia	12.19
New Zealand	13.4
Papua New Guinea	24.89

- (A) Explain why the calculation below is not a correct method for finding the birth rate per 1000 for Australasia as a whole.

$$\frac{12.19 + 13.4 + 24.89}{3} \approx 16.83$$

[1]

- (B) Without doing any calculations, explain whether the birth rate per 1000 for Australasia as a whole is higher or lower than 16.83. [1]

The scatter diagram in Fig. 9.2 shows birth rate per 1000 and physicians/1000 population for all the countries in the pre-release data set.

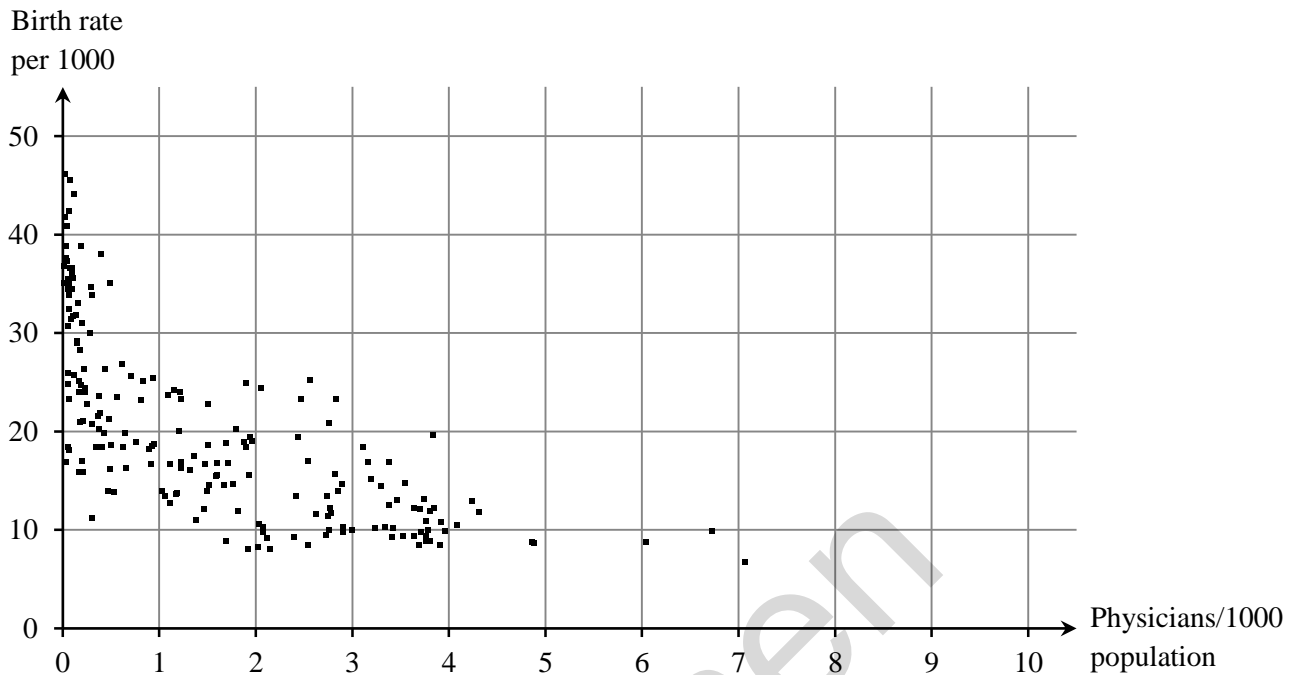


Fig. 9.2

(iii) Describe the correlation in the scatter diagram. [1]

(iv) Discuss briefly whether the scatter diagram shows that high birth rates would be reduced by increasing the number of physicians in a country. [1]

10 A company operates trains. The company claims that 92% of its trains arrive on time. You should assume that in a random sample of trains, they arrive on time independently of each other.

(i) Assuming that 92% of the company's trains arrive on time, find the probability that in a random sample of 30 trains operated by this company

(A) exactly 28 trains arrive on time, [2]

(B) more than 27 trains arrive on time. [2]

A journalist believes that the percentage of trains operated by this company which arrive on time is lower than 92%.

(ii) To investigate the journalist's belief a hypothesis test will be carried out at the 1% significance level. A random sample of 18 trains is selected. For this hypothesis test,

- state the hypotheses,
- find the critical region.

[5]

11 In this question you must show detailed reasoning.

Fig. 11 shows the curve $y = f(x)$, where $f(x)$ is a cubic function. The coordinates of the turning points and the points of intersection with the axes are shown in Fig. 11.

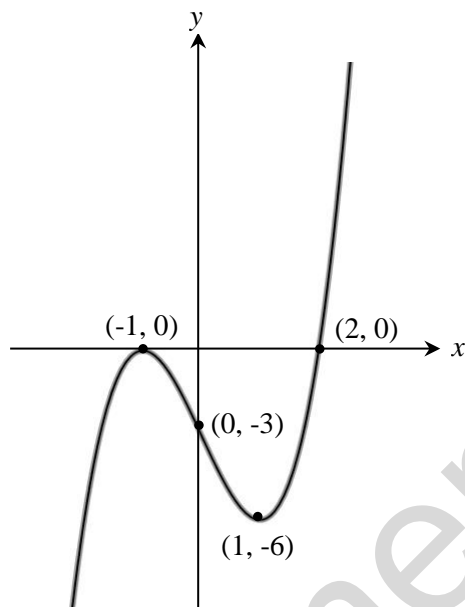


Fig. 11

Show that the tangent to $y = f(x)$ at $x = t$ is parallel to the tangent to $y = f(x)$ at $x = -t$ for all values of t . [6]

12 Given that $\arcsin x = \arccos y$, prove that $x^2 + y^2 = 1$. [Hint: Let $\arcsin x = \theta$] [3]

END OF QUESTION PAPER

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Qu 9: Central Intelligence Agency, The World Factbook 2013-14. Washington, DC, 2013 . <https://www.cia.gov/library/publications/the-world-factbook/index.html>

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