**Formulae**
**A Level Mathematics A (H240)**

**Arithmetic series**

$$S_n = \tfrac{1}{2}n(a+l) = \tfrac{1}{2}n\{2a+(n-1)d\}$$

**Geometric series**

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \quad \text{for } |r| < 1$$

**Binomial series**

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \ldots + {}^nC_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N})$$

$$\text{where } {}^nC_r = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1, n \in \mathbb{R})$$

**Differentiation**

| $f(x)$ | $f'(x)$ |
|---|---|
| $\tan kx$ | $k\sec^2 kx$ |
| $\sec x$ | $\sec x \tan x$ |
| $\cot x$ | $-\operatorname{cosec}^2 x$ |
| $\operatorname{cosec} x$ | $-\operatorname{cosec} x \cot x$ |

Quotient rule $y = \dfrac{u}{v}, \ \dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

**Differentiation from first principles**

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

**Integration**

$$\int \frac{f'(x)}{f(x)}\, dx = \ln|f(x)| + c$$

$$\int f'(x)(f(x))^n\, dx = \frac{1}{n+1}(f(x))^{n+1} + c$$

Integration by parts $\displaystyle \int u\frac{dv}{dx}\, dx = uv - \int v\frac{du}{dx}\, dx$

Small angle approximations

$\sin\theta \approx \theta$, $\cos\theta \approx 1 - \tfrac{1}{2}\theta^2$, $\tan\theta \approx \theta$ where $\theta$ is measured in radians

## Trigonometric identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad \left(A \pm B \neq \left(k + \tfrac{1}{2}\right)\pi\right)$$

## Numerical methods

Trapezium rule: $\int_a^b y \, dx \approx \tfrac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $f(x) = 0$: $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

## Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A)P(B \mid A) = P(B)P(A \mid B) \quad \textbf{or} \quad P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

## Standard deviation

$$\sqrt{\frac{\sum(x - \bar{x})^2}{n}} = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2} \quad \textbf{or} \quad \sqrt{\frac{\sum f(x - \bar{x})^2}{\sum f}} = \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2}$$

## The binomial distribution

If $X \sim B(n, p)$ then $P(X = x) = \binom{n}{x}p^x(1 - p)^{n-x}$, Mean of $X$ is $np$, Variance of $X$ is $np(1 - p)$

## Hypothesis test for the mean of a normal distribution

If $X \sim N(\mu, \sigma^2)$ then $\bar{X} \sim N\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$

## Percentage points of the normal distribution

If $Z$ has a normal distribution with mean 0 and variance 1 then, for each value of $p$, the table gives the value of $z$ such that $P(Z \leqslant z) = p$.

| $p$ | 0.75 | 0.90 | 0.95 | 0.975 | 0.99 | 0.995 | 0.9975 | 0.999 | 0.9995 |
|---|---|---|---|---|---|---|---|---|---|
| $z$ | 0.674 | 1.282 | 1.645 | 1.960 | 2.326 | 2.576 | 2.807 | 3.090 | 3.291 |

## Kinematics

Motion in a straight line

$$v = u + at$$

$$s = ut + \tfrac{1}{2}at^2$$

$$s = \tfrac{1}{2}(u + v)t$$

$$v^2 = u^2 + 2as$$

$$s = vt - \tfrac{1}{2}at^2$$

Motion in two dimensions

$$\mathbf{v} = \mathbf{u} + \mathbf{a}t$$

$$\mathbf{s} = \mathbf{u}t + \tfrac{1}{2}\mathbf{a}t^2$$

$$\mathbf{s} = \tfrac{1}{2}(\mathbf{u} + \mathbf{v})t$$

$$\mathbf{s} = \mathbf{v}t - \tfrac{1}{2}\mathbf{a}t^2$$