OCR
Oxford Cambridge and RSA

…day June 20XX – Morning/Afternoon

A Level Mathematics A

H240/03 Pure Mathematics and Mechanics

SAMPLE MARK SCHEME

Duration: 2 hours

MAXIMUM MARK 100

This document consists of 20 pages

Text Instructions

1. Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

2. Subject-specific Marking Instructions for A Level Mathematics A

a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
Mark for explaining a result or establishing a given result. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep*’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. There is no penalty for using a wrong value for g. E marks will be lost except when results agree to the accuracy required in the question.

g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate’s data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. ‘Fresh starts’ will not affect an earlier decision about a misread. Note that a miscopy of the candidate’s own working is not a misread but an accuracy error.

i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AO	Guidance	
1	**(i)**		5 Substituting $x=-3$ into $\lvert 2x-1\rvert$ 7	**B1** **M1** **A1** **[3]**	**1.1** **1.1a** **1.1**		
1	**(ii)**		$2x-1>x+1$ therefore $x>2$ $-(2x-1)>x+1$ (Allow ± in bracket) $x<0$ $\{x:x<0\}\cup\{x:x>2\}$	**B1** **M1** **A1** **A1** **[4]**	**1.1** **3.1a** **1.1** **2.5**	**OR** **B1** for a sketch of $y=\lvert 2x-1\rvert$ and $y=x+1$ on the same axes **M1** attempt to find the points of intersection **A1** obtain $x>2$ and $x<0$ **A1** $\{x:x<0\}\cup\{x:x>2\}$	**OR** **B1** $(2x-1)^2>(x+1)^2$ seen **M1** attempt to multiply out and simplify, then solve quadratic **A1** obtain $x>2$ and $x<0$ **A1** $\{x:x<0\}\cup\{x:x>2\}$
2	**(i)**		$\frac{0.25}{2}\left(1+0.7071+2\left(0.970+0.8944+0.8\right)\right)$ 0.880	**B1** **M1** **A1** **[3]**	**1.1** **1.1a** **1.1**	Obtain all five ordinates and no others: 0.7071, 0.8944, 1, 0.8, 0.970 Use correct structure for trapezium rule with $h=0.25$ 0.880 or better (0.87953077)	Accept exact values: 1, $\frac{4}{\sqrt{17}}$, $\frac{2}{\sqrt{5}}$, $\frac{4}{5}$, $\frac{1}{\sqrt{2}}$ *x*-coordinates used **M0**. Omission of large brackets unless implied by correct answer **M0** Accept 0.88 (0.87953077)
2	**(ii)**		"Use smaller intervals" or " use more trapezia"	**B1** **[1]**	**2.4**		

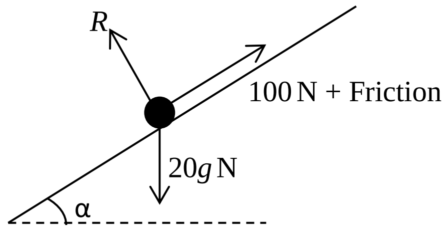
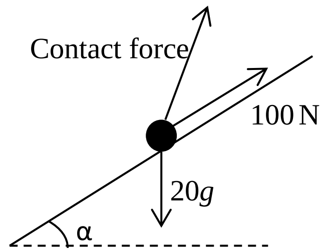
Question			Answer	Marks	AO	Guidance	
3			**DR** $5\sin 2x = 3\cos x \Rightarrow 10\sin x\cos x = 3\cos x$	**B1**	1.1	Use $\sin 2x = 2\sin x\cos x$ to obtain correct identity	**SC2** For use of identity followed by cancelling $\cos x$, leading to $\sin x = \frac{3}{10}$.
			$\cos x(10\sin x - 3) = 0$	**M1**	1.1a	Attempt to factorise	
			$\cos x \neq 0$ for $0^\circ < x < 90^\circ$	**E1**	2.1		
			so $\sin x = \frac{3}{10}$	**A1**	1.1		
				[4]			
4			When θ is small $1 + \cos\theta - 3\cos^2\theta$ $\approx 1 + \left(1 - \frac{1}{2}\theta^2\right) - 3\left(1 - \frac{1}{2}\theta^2\right)^2$	**M1**	1.1a	Attempt to use $\cos\theta \approx 1 - \frac{1}{2}\theta^2$ or $= 1 + \left(1 - \frac{1}{2}\theta^2 + ...\right)$ $-3\left(1 - \frac{1}{2}\theta^2 + ...\right)^2$	**OR** **M1** Attempt to use $\cos\theta \approx 1 - \frac{1}{2}\theta^2$
			$= 1 + \left(1 - \frac{1}{2}\theta^2\right) - 3\left(1 - \theta^2 + \frac{1}{4}\theta^4\right)$ $= 1 + 1 - \frac{1}{2}\theta^2 - 3 + 3\theta^2 - \frac{3}{4}\theta^4$	**M1**	1.1	Multiply out	**M1** use trigonometric identity $1 + \cos\theta - 3\cos^2\theta$ $= 1 + \cos\theta - \frac{3}{2} - \frac{3}{2}\cos 2\theta$
			Since θ is small, we can neglect the higher order terms	**E1**	2.5	For explanation of loss of θ^4 term and consistent use of notation throughout (Working need not be fully correct)	**E1** For showing clearly which identity has been used and consistent use of notation throughout
			so $1 + \cos\theta - 3\cos^2\theta \approx -1 + \frac{5}{2}\theta^2$ as required	**E1**	2.1	AG Clearly obtained www Condone θ^4 term missing without explanation and inconsistent notation	**E1** AG Clearly obtained www Condone inconsistent notation
				[4]			

Question			Answer	Marks	AO	Guidance	
5	(i)		Obtain $1+\frac{1}{3}px$	B1	1.1		
			$\left(\frac{1}{2}\right)\left(\frac{1}{3}\right)\left(\frac{-2}{3}\right)(px)^2$	M1	1.1		Attempt the x^2 term at least in the form ${}^6C_2kx^2$
			Obtain $1+\frac{1}{3}px-\frac{1}{9}p^2x^2$	A1	1.1	Must be simplified	
				[3]			
5	(ii)		$(1+qx)\left(1+\frac{1}{3}px-\frac{1}{9}p^2x^2\right)$ $=1+\left(\frac{1}{3}p+q\right)x+\left(\frac{1}{3}pq-\frac{1}{9}p^2\right)x^2$	M1	3.1a		Expand $(1+qx)$ and their $1+\frac{1}{3}px-\frac{1}{9}p^2x^2$ and compare coefficients
			$\frac{1}{3}p+q=1$ (*) $\frac{1}{3}pq-\frac{1}{9}p^2=-\frac{2}{9}$	M1	3.1a	Obtain two equations in *p* and *q* and show evidence of substitution for *p* or *q* to obtain an equation in one variable	
			$2p^2-3p-2=0$	M1	1.1	Solve a 3 term quadratic equation in a single variable.	Or $18q^2-27q+7=0$ Solve their quadratic
			$p=2$ or $-\frac{1}{2}$	A1	1.1	Obtain any two values	
			$q=\frac{1}{3}$ or $\frac{7}{6}$	A1FT	1.1	Obtain all 4 values , or FT their *p* and (*)	with indication of correct pairings
				[5]			

Question			Answer	Marks	AO	Guidance	
6			$\frac{dy}{dx}=2x+k+4x^{-2}$	M1	1.1a	Attempt to differentiate	Power decreases by 1 for at least 2 terms
			$2(-2)+k+4(-2)^{-2}=0$	M1	3.1a	Substitute $x=-2$, equate to 0 and attempt to solve	
			$k=3$	A1	1.1		
			$\frac{d^2y}{dx^2}=2-8x^{-3}$				
			$2-8x^{-3}=0$	M1	3.1a	Equate second derivative to 0 and attempt to solve	
			$x=4^{\frac{1}{3}}$	A1	1.1		
			for $x<4^{\frac{1}{3}} \Rightarrow \frac{d^2y}{dx^2}<0$ for $x>4^{\frac{1}{3}} \Rightarrow \frac{d^2y}{dx^2}>0$	E1	2.1	Consider convex/concave either side of $x=4^{\frac{1}{3}}$ and conclude	
			When $x=4^{\frac{1}{3}}, \frac{dy}{dx}\neq 0$ hence not a stationary point	E1	2.1	Consider gradient at $x=4^{\frac{1}{3}}$, or justify that $x=-2$ is the only stationary point	
				[7]			

Question			Answer	Marks	AO	Guidance	
7	(i)		$u = x^2 + 1$ $\mathrm{d}u = 2x\mathrm{d}x$	**M1**	**1.1a**	Attempt a substitution of x and $\mathrm{d}x$	**M0** for $\mathrm{d}u = \mathrm{d}x$
			$\frac{5}{2}\int (u-1)u^{\frac{1}{2}}\mathrm{d}u$	**M1**	**1.1**	Replace as far as $k\int (u-1)u^{\frac{1}{2}}du$	
			$\frac{5}{2}\int \left(u^{\frac{3}{2}} - u^{\frac{1}{2}}\right)\mathrm{d}u$	**A1**	**1.1**		
			$u^{\frac{5}{2}} - \frac{5}{3}u^{\frac{3}{2}} + c$	**M1**	**1.1**	Integrate their integral if in u	
			$\left(x^2+1\right)^{\frac{5}{2}} - \frac{5}{3}\left(x^2+1\right)^{\frac{3}{2}} + c$	**A1**	**1.1**	Do not condone missing $+c$ in both **(i)** and **(ii)**	
				[5]			
7	(ii)		$\int \tan^2\theta\,\mathrm{d}\theta = \int\left(\sec^2\theta - 1\right)\mathrm{d}\theta$	**M1**	**1.1**	Award for sight of the intermediate result	**OR** **M1** $\int \theta\tan^2\theta\,\mathrm{d}\theta = \int \theta\left(\sec^2\theta - 1\right)\mathrm{d}\theta$
			$= \tan\theta - \theta$	**A1**	**1.1**		**A1** $= \int \theta\sec^2\theta\,\mathrm{d}\theta - \int \theta\,\mathrm{d}\theta$
			$u = \theta, \mathrm{d}v = \tan^2\theta$	**M1**	**3.1a**	Recognise integration by parts with appropriate choice of u and $\mathrm{d}v$	**M1** $u = \theta,\ dv = \sec^2\theta$
			So $\int \theta\tan^2\theta\,\mathrm{d}\theta = \theta(\tan\theta - \theta) - \int(\tan\theta - \theta)\,\mathrm{d}\theta$	**A1**	**1.1**	Obtain correct intermediate result	**A1** So $\int \theta\tan^2\theta\,\mathrm{d}\theta$ $= \theta\tan\theta - \int \tan\theta\,\mathrm{d}\theta - \frac{1}{2}\theta^2$
			$-\frac{1}{2}\theta^2 + \theta\tan\theta - \ln\lvert\sec\theta\rvert + c$	**A1**	**1.1**		**A1** $= -\frac{1}{2}\theta^2 + \theta\tan\theta - \ln\lvert\sec\theta\rvert + c$
				[5]			

Question			Answer	Marks	AO	Guidance	
8	(i)		**DR** $BE=\sqrt{3}$ from the standard triangle BDE $BC = AB\cos 45$ $BC=\frac{1+\sqrt{3}}{\sqrt{2}}=\frac{\sqrt{2}+\sqrt{6}}{2}$	B1 M1 E1 **[3]**	2.2a 2.1 2.2a	Or $AB=1+\sqrt{3}$ seen oe or Pythagoras' theorem AG	**B0** for decimal Must be seen $\frac{1+\sqrt{3}}{\sqrt{2}}$ must be seen
8	(ii)		**DR** Triangle ABC is isosceles so $BC = AC$ but $AC = CD+\sqrt{2}$ so $CD=\frac{\sqrt{2}+\sqrt{6}}{2}-\sqrt{2}$ $=\frac{\sqrt{6}-\sqrt{2}}{2}$ $\sin 15=\frac{CD}{BD}=\frac{\sqrt{6}-\sqrt{2}}{2}\div 2=\frac{\sqrt{6}-\sqrt{2}}{4}$	B1 M1 A1 **[3]**	2.4 2.1 2.2a	State or imply that $BC = AC$ and state $AC = CD+\sqrt{2}$ Obtain expression for *CD*, may be unsimplified Obtain expression for $\sin 15$ and simplify to answer given	**M0** if decimals seen **SC1** for showing using addition formula
9	(i)		Attempt resolution of forces Horizontal component $=5+2\cos 40\ (=6.5321)$ Vertical component $=2\sin 40\ (=1.2856)$ $\sqrt{6.5321^2+1.2856^2}=6.66\text{ N}$	M1 A1 A1 **[3]**	1.1a 1.1 1.1	Allow sin/cos confusion Allow for either the horizontal or vertical component correct Use correct method for magnitude	**OR** **M1** Form triangle of forces **A1** Use cosine rule with 140° **A1** Obtain 6.66 N
9	(ii)		$\tan^{-1}\left(\frac{2\sin 40}{5+2\cos 40}\right)=11.1^\circ$	B1FT **[1]**	1.1	**FT** their components from part (i)	

Question			Answer	Marks	AO	Guidance	
10	**(i)**			**B1**	**2.1**	Any equivalent which makes clear the relationships between: Reaction, 100 N force, friction acting upwards, weight of $20g$ N A diagram is not *necessary* provided that sufficient explanation is given.	**OR** 
			Resolve parallel to the slope: $100 + F - 20g\sin\alpha = 0$ (*)	**M1**	**3.3**		
			Resolve perpendicular to the slope and friction force is maximum: $R = 20g\cos\alpha$ and $F = \mu R$	**M1**	**3.3**		
			Substitute and obtain $20g\sin\alpha = 20g\mu\cos\alpha + 100$	**E1** **[4]**	**1.1**	AG	
10	**(ii)**		All forces shown on diagram of inclined plane			Reaction, 150 N force, friction acting downwards, weight of $20g$ N	
			Resolve parallel to the slope: $150 - F - 20g\sin\alpha = 0$ (**)	**B1**	**3.3**		
			From * and ** $250 - 40g\sin\alpha = 0$	**M1**	**3.4**	Eliminate μ and attempt to solve for α.	One valid step after elimination required
			$\alpha = \sin^{-1}\dfrac{25}{4g}$	**A1** **[3]**	**1.1**		

Question			Answer	Marks	AO	Guidance	
11	(i)		$\mathbf{v} = 6t^2\mathbf{i} + (10t - 4)\mathbf{j}$ $\mathbf{v} = 2.94\mathbf{i} + 3\mathbf{j}$ $90 - \tan^{-1}\left(\frac{2.94}{3}\right)$ $= 044^\circ$	B1 M1 A1 [3]	1.1 3.1a 1.1	At least one term reduces in power by 1 Substitution of $t = 0.7$, use $\tan^{-1}\left(\frac{y}{x}\right)$ and obtain $90 - 45.578 = 44.4^\circ$ to give a 3 figure bearing	For a complete method to find a bearing
11	(ii)		$\mathbf{a} = 12t\mathbf{i} + 10\mathbf{j}$ $\mathbf{a} = 8.4\mathbf{i} + 10\mathbf{j}$ Use $\mathbf{F} = m\mathbf{a}$ and use Pythagoras Obtain 1.57 N	M1 A1 M1 A1FT [4]	1.1 1.1 3.3 3.4	Attempt differentiation of **v** Substitute $t = 0.7$ FT their **a** at t = 0.7	
11	(iii)		$6t^2 = 10t - 4$ $6t^2 - 10t + 4 = 0$ so $t = 1$ or $\frac{2}{3}$ E.g. **i** component always positive so both values are valid	M1 E1 [2]	2.2a 2.3	Equate **i** and **j** components and solve FT their **v** from part (i) if it leads to a quadratic BC Must include comment on why equating components is sufficient in this case.	

Question			Answer	Marks	AO	Guidance	
12	**(i)**	**(a)**	Vertical component of $U = 10\sin 40$	**B1**	**1.1**		
			Vertical component of velocity = $10\sin 40 - gt = 0$	**M1**	**3.3**	Use $v = u - gt$ with $v = 0$ Allow sign error or sin/cos confusion	
			Obtain $t = 0.656$	**A1**	**1.1**		0.6559057242…
			Vertical displacement = $10\sin 40t - \frac{1}{2}gt^2\ (+c)$	**M1**	**3.4**	Use $s = ut + \frac{1}{2}gt^2$ or $s = \int v\,dt$	Allow if initial height not seen **M1** may be awarded if seen in part **(i)(b)**
			Obtain 2.11 + 1.5 = 3.61 m	**A1FT** **[5]**	**1.1**	FT their "2.11" + 1.5	3.608040363…
12	**(i)**	**(b)**	Horizontal component of $U = 10\cos 40$	**B1**	**1.1**	Use the horizontal component of U	Allow $10\sin 40$ if $10\cos 40$ given in part (i)
			$6 = 10\cos 40t$	**M1**	**3.3**	Attempt horizontal resolution equated to 6 Allow sin/cos error	
			$t = 0.783$	**A1**	**1.1**		0.7832443736…
			$(2.028586218 + 1.5) - 2.5 = 1.03\text{ m}$	**A1** **[4]**	**3.4**	Substitute t in $10\sin 40t - \frac{1}{2}gt^2\ (+1.5)$ and subtract 2.5	
12	**(ii)**		Use $1 = 6\tan 40 - \frac{(9.8)6^2\sec^2 40}{2U^2}$	**M1**	**3.1b**	Use $y = x\tan\theta - \frac{gx^2\sec^2\theta}{2U^2}$ with $x = 6$ and $\theta = 40$	Allow $y = 2.5$ for **M1**
			$U^2 = 74.5....$	**M1**	**1.1**	Attempt to make U the subject	**OR** BC
			Obtain $U = 8.63$	**A1** **[3]**	**1.1**	BC	8.631677404…